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Energy Momentum Distributions of Texture and Monopole Metrics in General Relativity

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Abstract The aim of this study is to investigate the energy-momentum distributions of texture and monopole topological defects metrics in general relativity (GR). For this aim Einstein, Bergmann-Thomson, Landau-Lifshitz (LL), Møller and Papapetrou energy-momentum densities have been used in general relativity theory. We obtained that (i) for the texture metric only Einstein and Bergmann-Thomson energy densities give the same results but the others energy and momentum densities do not provide the same results in GR; (ii) for the monopole metric, while Einstein, Bergmann-Thomson and Papapetrou energy and momentum densities are giving the same energy-momentum results, Møller and Landau-Lifshitz densities do not give the same energy results with the other definitions in GR.

Keywords Texture · Monopole · Energy-momentum density

1 Introduction

In the various gravitation theories like as general relativity and the tetrad theory of gravity (teleparallel gravity) some researchers considered energy-momentum problem for different models. This topic is still important question in the gravitation theories. First study about energy-momentum localization has been madden by Einstein [1]. After Einstein, various authors have represented energy-momentum distributions for example Tolman [2], Papapetrou [3], Landau-Liftshitz [4], Bergmann-Thomson [5], Møller [6], Weinberg [7] also the teleparallel gravity (TG) versions of the Møller [8], Einstein, Landau-Lifshitz and Bergmann-Thomson prescriptions [9, 10]. According to Virbhadra; except the Møller energy-momentum definition, other energy-momentum complexes give the expressive solutions when we transform the space-time metric into the quasi-Cartesian coordinates [10] also Møller energy and momentum complex can calculation in any coordinate system [10–12].

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Various authors have tried to purpose energy-momentum problem using the different metrics and different energy-momentum distributions and they obtained some interesting results in general relativity and teleparallel gravity [10, 13–22]. According to Xulu [23] Virbhadra and his collaborators have showed that different energy-momentum complexes give same and reasonable results for a given space-time [10, 24–26]. Radinschi [27] using the energy momentum complexes of Tolman, Bergmann-Thomson, Møller, Einstein, Landau-Lifshitz in general relativity, showed that total energy is zero everywhere. Xulu [23, 28] has discussed Einstein and Møller energy-momentum definitions for the spherically non-static symmetric metric and obtained different results for this model. Also, Xulu [21] investigated Weinberg, Papapetrou and Landau-Lifshitz energy-momentum definitions in Bianchi I model and he found that the total energy is zero everywhere for this model also, this study agrees with the study of Banerjee-Sen. Aygün and Yilmaz [29] have investigated the topological defect metrics i.e monopoles and textures using the Møller energy distribution in spherically coordinates for general relativity and teleparallel gravity and they found zero energy densities for these metrics.

The layout of the paper is as follows. In Sect. 2, we introduce the textures and the transformation of this metric. Then we give the energy-momentum definitions of the Einstein, Bergmann-Thomson, Landau-Lifshitz, Møller and Papapetrou in general theory of relativity and related calculations. In Sect. 3, we introduce the monopoles and the transformation of this metric. Then we have calculated of the Einstein, Bergmann-Thomson, Landau-Lifshitz, Møller and Papapetrou energy-momentum distributions of monopole in general theory of relativity. Finally, we summarize and discuss our results. Throughout this paper all indices run 0 to 3.

2 Some Energy Momentum Distributions of Textures in GR

The texture metric is given by [30]

$$ds^2 = -dt^2 + dr^2 + r^2(1+m)[d\theta^2 + \sin^2\theta d\phi^2] \tag{1}$$

where m is the function of r and t only. It is well known that the energy-momentum complexes give meaningful results if calculations are performed in quasi-Cartesian coordinates [31]. The line element in (1) can be transformed to quasi-Cartesian coordinates:

$$\begin{aligned} ds^2 = -dt^2 &+ \frac{r^2+m(y^2+z^2)}{r^2}dx^2 + \frac{r^2+m(x^2+z^2)}{r^2}dy^2 \\ &+ \frac{r^2+m(x^2+y^2)}{r^2}dz^2 - \frac{2m}{r^2}(xydxdy + xzdxdz + yzdydz) \end{aligned} \tag{2}$$

where the coordinates r, θ, ϕ, t in (1) and x, y, z, t are related through

$$\begin{aligned} &r = \sqrt{x^2+y^2+z^2}, \qquad \theta = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \\ &\phi = \arctan\left(\frac{y}{x}\right), \quad t = t \end{aligned} \tag{3}$$

In this section, we will calculate the Einstein, Bergmann-Thomson, LL, Møller and Papapetrou energy-momentum distributions in GR for the texture metric, respectively.

(i) *Einstein Energy Momentum Distribution of Texture Metric*: The Einstein energy-momentum complex [1] is given by

$$\S_i^k = \frac{1}{16\pi}\chi_{i,l}^{kl} \tag{4}$$

where the Einstein's superpotential χ_i^{kl} is of the form

$$\chi_i^{kl} = \frac{g_{in}}{\sqrt{-g}}[-g(g^{kn}g^{lm} - g^{ln}g^{km})]_{,m} \tag{5}$$

$\S_0^0$ and $\S_\alpha^0$ are the energy and momentum density components. To obtain energy and momentum densities in Einstein's complex associated with the space-time in (2), we evaluate the required components of χ_i^{kl} (5) as following

$$\begin{aligned}
&\chi_1^{01} = \frac{m_t(x^2+r^2)}{r^2}, \qquad \chi_1^{02} = \chi_2^{01} = \frac{xym_t}{r^2}, \qquad \chi_1^{03} = \frac{xzm_t}{r^2} \\
&\chi_2^{02} = \frac{m_t(y^2+r^2)}{r^2}, \qquad \chi_2^{03} = \chi_3^{02} = \frac{yzm_t}{r^2}, \qquad \chi_3^{03} = \frac{m_t(z^2+r^2)}{r^2} \\
&\chi_0^{01} = \frac{2x(m+1)(m+xm_x) + x(ym_y+zm_z)(2m+1) + m_x(y^2+z^2)}{(m+1)r^2} \\
&\chi_0^{02} = \frac{2y(m+1)(m+ym_y) + y(xm_x+zm_z)(2m+1) + m_y(z^2+x^2)}{(m+1)r^2} \\
&\chi_0^{03} = \frac{2z(m+1)(m+zm_z) + z(ym_y+xm_x)(2m+1) + m_z(x^2+y^2)}{(m+1)r^2}
\end{aligned} \tag{6}$$

Here *x,y,z,t* indices represent the first derivatives of *x,y,z,t*, respectively. Using these components in (4), we get the Einstein energy and momentum densities for texture metric as following

$$\begin{aligned}
\S_0^0 = {} & \frac{1}{16\pi r^2(1+m)^2}\{m_{xx}(4x^2m + 2x^2m^2 + y^2m + z^2m + r^2 + x^2) \\
& + m_{yy}(4y^2m + 2y^2m^2 + x^2m + z^2m + r^2 + y^2) \\
& + m_{zz}(4z^2m + 2z^2m^2 + x^2m + y^2m + r^2 + z^2) \\
& + m_x(10mx + 4x + 6xm^2 + 2xym_y + 2xzm_z - m_xz^2 - m_xy^2) \\
& + m_y(10my + 4y + 6ym^2 + 2yzm_z - m_yz^2 - m_yx^2) \\
& + m_z(10mz + 4z + 6zm^2 - m_zx^2 - m_zy^2) + (2 + 4m^2 + 6m) \\
& \times (zym_{yz} + xzm_{xz} + xym_{xy}) + 2m(1 + 2m + m^2)\} \\
\S_1^0 = {} & \frac{1}{16\pi r^2}\{m_{xt}(r^2+x^2) + x(2m_t + zm_{zt} + ym_{yt})\} \\
\S_2^0 = {} & \frac{1}{16\pi r^2}\{m_{yt}(r^2+y^2) + y(2m_t + zm_{zt} + xm_{xt})\} \\
\S_3^0 = {} & \frac{1}{16\pi r^2}\{m_{zt}(r^2+z^2) + z(2m_t + ym_{yt} + xm_{xt})\}
\end{aligned} \tag{7}$$

(ii) *Bergmann-Thomson Energy Momentum Distribution of Texture Metric*: The energy-momentum complex of Bergmann-Thomson is given by [5]

$$\Xi^{\mu\nu} = \frac{1}{16\pi} \Pi^{\mu\nu\alpha}_{,\alpha} \tag{8}$$

where

$$\Pi^{\mu\nu\alpha} = g^{\mu\beta} V_\beta^{\nu\alpha} \tag{9}$$

with

$$V_\beta^{\nu\alpha} = -V_\beta^{\alpha\nu} = \frac{g_{\beta\xi}}{\sqrt{-g}}[-g(g^{\nu\xi}g^{\alpha\rho} - g^{\alpha\xi}g^{\nu\rho})]_{,\rho} \tag{10}$$

here Ξ_0^0 is the energy density, Ξ_μ^0 are the momentum density components, and Ξ_0^μ are the components of the energy current density. To obtain energy and momentum densities in Bergmann-Thomson's complex associated with the texture metric, we evaluate the required components of $\Pi^{\mu\nu\alpha}$ (9) as following

$$\begin{aligned}
&\Pi^{101} = \frac{m_t(x^2 + r^2 + 2mx^2)}{r^2(m+1)}, \qquad \Pi^{102} = \frac{xym_t(2m+1)}{r^2(m+1)} \\
&\Pi^{103} = \Pi^{301} = \frac{xzm_t(2m+1)}{r^2(m+1)}, \qquad \Pi^{202} = \frac{m_t(y^2 + r^2 + 2my^2)}{r^2(m+1)} \\
&\Pi^{203} = \Pi^{302} = \frac{yzm_t(2m+1)}{r^2(m+1)}, \qquad \Pi^{303} = \frac{m_t(z^2 + r^2 + 2mz^2)}{r^2(m+1)} \\
&\Pi^{001} = -\frac{m_x(x^2 + r^2) + x(ym_y + zm_z)(2m+1) + 2xm(m+1+xm_x)}{r^2(1+m)} \\
&\Pi^{002} = -\frac{m_y(y^2 + r^2) + y(xm_x + zm_z)(2m+1) + 2ym(m+1+ym_y)}{r^2(1+m)} \\
&\Pi^{003} = -\frac{m_z(z^2 + r^2) + z(xm_x + ym_y)(2m+1) + 2zm(m+1+zm_z)}{r^2(1+m)}
\end{aligned} \tag{11}$$

Using these components in (8), we get the Bergmann-Thomson energy and momentum densities for texture metric as following

$$\begin{aligned}
\Xi_0^0 = {} & \frac{1}{16\pi r^2(1+m)^2}\{m_{xx}(4x^2m + 2x^2m^2 + y^2m + z^2m + r^2 + x^2) \\
& + m_{yy}(4y^2m + 2y^2m^2 + x^2m + z^2m + r^2 + y^2) \\
& + m_{zz}(4z^2m + 2z^2m^2 + x^2m + y^2m + r^2 + z^2) \\
& + m_x(10mx + 4x + 6xm^2 + 2xym_y + 2xzm_z - m_xz^2 - m_xy^2) \\
& + m_y(10my + 4y + 6ym^2 + 2yzm_z - m_yz^2 - m_yx^2) \\
& + m_z(10mz + 4z + 6zm^2 - m_zx^2 - m_zy^2) + (2 + 4m^2 + 6m) \\
& \times (zym_{yz} + xzm_{xz} + xym_{xy}) + 2m(1 + 2m + m^2)\} \\
\Xi_1^0 = {} & \frac{1}{16\pi r^2(1+m)}\{(m+1)(m_{xt}(r^2 + x^2) + x(ym_{yt} + zm_{zt})) \\
& - m_xm_t(y^2 + z^2) + xm_t(zm_z + ym_y + 2 + 2m)\}
\end{aligned} \tag{12}$$

$$\Xi_2^0 = \frac{1}{16\pi r^2(1+m)}\{(m+1)(m_{yt}(r^2+y^2)+y(xm_{xt}+ym_{zt})) - m_y m_t(x^2+z^2)+ym_t(xm_x+zm_z+2+2m)\}$$
$$\Xi_3^0 = \frac{1}{16\pi r^2(1+m)}\{(m+1)(m_{zt}(r^2+z^2)+z(xm_{xt}+ym_{yt})) - m_z m_t(x^2+y^2)+zm_t(xm_x+ym_y+2+2m)\}$$

(iii) *Landau-Lifshitz Energy Momentum Distribution of Texture Metric*: The energy-momentum complex of Landau-Lifshitz [4] is given by

$$\mathcal{L}^{ij} = \frac{1}{16\pi}\Upsilon^{ikjl}{}_{,kl} \tag{13}$$

where Υ^{ikjl} with symmetries of the Riemann tensor and is defined by

$$\Upsilon^{ikjl} = -g(g^{ij}g^{kl} - g^{il}g^{kj}) \tag{14}$$

The quantity $\mathcal{L}_0^0$ represents the energy density and $\mathcal{L}_\alpha^0$ represents the components of the total momentum density. In order to evaluate the energy and momentum densities in Landau-Lifshitz's prescription associated with the texture metric (2), we evaluate the required components of Υ^{ikjl} (14);

$$\begin{aligned}
\Upsilon^{1010} &= -\Upsilon^{1001} = -\frac{(mx^2+r^2)(m+1)}{r^2}\\
\Upsilon^{1002} &= -\Upsilon^{0102} = \frac{mxy(m+1)}{r^2}\\
\Upsilon^{2003} &= \Upsilon^{3002} = \frac{myz(m+1)}{r^2}\\
\Upsilon^{1003} &= \Upsilon^{3001} = \frac{mxz(m+1)}{r^2}\\
\Upsilon^{2002} &= \frac{(my^2+r^2)(m+1)}{r^2}\\
\Upsilon^{3003} &= \frac{(mz^2+r^2)(m+1)}{r^2}
\end{aligned} \tag{15}$$

Using these components in (13), we get the LL energy and momentum densities for texture as following,

$$\begin{aligned}
\mathcal{L}_0^0 &= \frac{1}{16\pi r^2}\{m_{xx}(x^2+r^2+2x^2m)+m_{yy}(y^2+r^2+2y^2m)\\
&\quad + m_{zz}(z^2+r^2+2z^2m)+(4m+2)(xym_{xy}+yzm_{yz}+xzm_{xz})\\
&\quad + 2xm_x(4m+2+xm_x)+2ym_y(4m+2+ym_y)\\
&\quad + 2zm_z(4m+2+zm_z)+4(yzm_ym_z+xym_xm_y+xzm_xm_z)+2m\}\\
\mathcal{L}_1^0 &= \frac{1}{16\pi r^2}\{(m+1)(r^2m_{xt}+x^2m_{xt}+xym_{yt}+xzm_{zt})\\
&\quad + 2xm_t(xm_x+ym_y+zm_z+2m+1)\}
\end{aligned} \tag{16}$$

$$\mathcal{L}_2^0 = \frac{1}{16\pi r^2}\{(m+1)(r^2 m_{yt} + y^2 m_{yt} + xy m_{xt} + yz m_{zt}) + 2y m_t(x m_x + y m_y + z m_z + 2m + 1)\}$$

$$\mathcal{L}_3^0 = \frac{1}{16\pi r^2}\{(m+1)(r^2 m_{zt} + z^2 m_{zt} + xz m_{xt} + yz m_{yt}) + 2z m_t(x m_x + y m_y + z m_z + 2m + 1)\}$$

(iv) *Møller Energy Momentum of Texture*: The energy-momentum complex of Møller [6] is given by

$$M_\mu^\nu = \frac{1}{8\pi}\lambda_{\mu,\alpha}^{\nu\alpha} \tag{17}$$

where the antisymmetric super-potential $\lambda_\mu^{\nu\alpha}$ is

$$\lambda_\mu^{\nu\alpha} = \sqrt{-g}[g_{\mu\beta,\gamma} - g_{\mu\gamma,\beta}]g^{\nu\gamma}g^{\alpha\beta} \tag{18}$$

and M_0^0 is the energy density and M_α^0 are the momentum density components. Using the metric in (2), we have found the required components of $\lambda_\mu^{\nu\alpha}$ (18);

$$\begin{aligned}
\lambda_1^{01} &= -\frac{m_t(y^2+z^2)}{r^2} \\
\lambda_2^{02} &= -\frac{m_t(x^2+z^2)}{r^2} \\
\lambda_1^{03} &= \lambda_3^{01} = \frac{xz m_t}{r^2} \\
\lambda_2^{03} &= \lambda_3^{02} = \frac{yz m_t}{r^2} \\
\lambda_3^{03} &= -\frac{m_t(y^2+x^2)}{r^2}, \quad \lambda_1^{02} = \frac{xy m_t}{r^2}
\end{aligned} \tag{19}$$

Using these components in (17), we get the Møller energy and momentum densities for texture metric as following,

$$\begin{aligned}
M_0^0 &= 0 \\
M_1^0 &= \frac{1}{8\pi}\frac{x(y m_{yt} + z m_{zt} + 2m_t) - m_{xt}(y^2+z^2)}{r^2} \\
M_2^0 &= \frac{1}{8\pi}\frac{y(x m_{xt} + z m_{zt} + 2m_t) - m_{yt}(x^2+z^2)}{r^2} \\
M_3^0 &= \frac{1}{8\pi}\frac{z(x m_{xt} + y m_{yt} + 2m_t) - m_{zt}(x^2+y^2)}{r^2}
\end{aligned} \tag{20}$$

(v) *Papapetrou Energy Momentum Distributing of Texture Metric*: The energy-momentum complex of Papapetrou is given by [3]

$$\Sigma^{\mu\nu} = \frac{1}{16\pi}N_{,\alpha\beta}^{\mu\nu\alpha\beta} \tag{21}$$

where

$$N^{\mu\nu\alpha\beta} = \sqrt{-g}(g^{\mu\nu}\eta^{\alpha\beta} - g^{\mu\alpha}\eta^{\nu\beta} + g^{\alpha\beta}\eta^{\mu\nu} - g^{\nu\beta}\eta^{\mu\alpha}) \tag{22}$$

Σ_0^0 is the energy density, Σ_μ^0 are the momentum density components, and Σ_0^μ are the components of energy current density. Using the line element (2), we have found the required components of $N^{\mu\nu\alpha\beta}$ (22);

$$\begin{aligned}
N^{1010} &= -N^{0011} = \frac{2r^2 + 2mx^2 + my^2 + mz^2}{r^2} \\
N^{2020} &= -N^{0022} = \frac{2r^2 + mx^2 + 2my^2 + mz^2}{r^2} \\
N^{3030} &= -N^{0033} = \frac{2r^2 + mx^2 + my^2 + 2mz^2}{r^2} \\
N^{1020} &= N^{2010} = -N^{0012} = \frac{xym}{r^2} \\
N^{1030} &= N^{3010} = -N^{0013} = \frac{xzm}{r^2} \\
N^{3020} &= \frac{zym}{r^2}.
\end{aligned} \tag{23}$$

Using these components in (21) we have obtained Papapetrou energy-momentum distribution for texture metric:

$$\begin{aligned}
\Sigma_0^0 &= \frac{1}{16\pi r^2}\{m_{xx}(r^2 + x^2) + m_{yy}(r^2 + y^2) + m_{zz}(r^2 + z^2) \\
&\quad + 2(xym_{xy} + xzm_{xz} + yzm_{yz} + m) + 4(xm_x + ym_y + zm_z)\} \\
\Sigma_1^0 &= \frac{m_{xt}(x^2 + r^2 + m(y^2 + z^2)) + (1 - m)x(ym_{yt} + zm_{zt}) + 2xm_t}{16\pi r^2} \\
\Sigma_2^0 &= \frac{m_{yt}(y^2 + r^2 + m(x^2 + z^2)) + (1 - m)y(xm_{xt} + zm_{zt}) + 2ym_t}{16\pi r^2} \\
\Sigma_3^0 &= \frac{m_{zt}(z^2 + r^2 + m(x^2 + y^2)) + (1 - m)z(xm_{xt} + ym_{yt}) + 2zm_t}{16\pi r^2}
\end{aligned} \tag{24}$$

3 Some Energy Momentum Distributions of Monopoles in GR

The monopole metric is given by [30],

$$ds^2 = -dt^2 + dr^2 + r^2(1 - 8\pi G\eta^2)[d\theta^2 + \sin^2\theta d\phi^2] \tag{25}$$

where η is a positive constant and describes the scale of symmetry breaking [30]. We obtain that the monopole metric in Cartesian coordinates with using (3) as follows,

$$\begin{aligned}
ds^2 &= -dt^2 + \frac{(r^2 - \Lambda(y^2 + z^2))dx^2}{r^2} + \frac{(r^2 - \Lambda(x^2 + z^2))dy^2}{r^2} \\
&\quad + \frac{(r^2 - \Lambda(x^2 + y^2))dz^2}{r^2} + \frac{2\Lambda}{r^2}(xydxdy + xzdxdz + yzdydz)
\end{aligned} \tag{26}$$

Here $\Lambda = 8\pi G\eta^2$. In this section, we will calculate the Einstein, Bergmann-Thomson, LL, Møller and Papapetrou energy-momentum distributions in GR for the monopole metric.

(i) *Einstein Energy Momentum Distribution of Monopole Metric*:

In order to calculate the energy and momentum densities in Einstein's complex associated with the monopole in (26), we evaluate the required components of χ_i^{kl} (5) as following

$$\chi_0^{01} = \frac{16\pi G\eta^2 x}{r^2}, \qquad \chi_0^{02} = \frac{16\pi G\eta^2 y}{r^2}, \qquad \chi_0^{03} = \frac{16\pi G\eta^2 z}{r^2} \tag{27}$$

Using these components in (4), we get the Einstein energy and momentum densities for monopole metric as follows

$$\begin{aligned} \S_0^0 &= \frac{G\eta^2}{r^2} \\ \S_1^0 &= \S_2^0 = 0 = \S_3^0 = 0 \end{aligned} \tag{28}$$

(ii) *Bergmann-Thomson Energy Momentum Distribution of Monopole Metric*:

In order to calculate the energy and momentum densities in Bergmann-Thomson's complex associated with the monopole metric, we evaluate the required components of $\Pi^{\mu\nu\alpha}$ (9) as follows

$$\begin{aligned} \Pi^{001} &= -\frac{16\pi G\eta^2 x}{r^2}, \qquad \Pi^{002} = -\frac{16\pi G\eta^2 y}{r^2} \\ \Pi^{003} &= -\frac{16\pi G\eta^2 z}{r^2} \end{aligned} \tag{29}$$

Using these components in (8), we get the Bergmann-Thomson energy and momentum densities for monopole metric as follows

$$\begin{aligned} \Xi_0^0 &= \frac{G\eta^2}{r^2} \\ \Xi_1^0 &= \Xi_2^0 = \Xi_3^0 = 0 \end{aligned} \tag{30}$$

(iii) *Landau-Lifshitz Energy Momentum Distribution of Monopole Metric*:

In order to evaluate the energy and momentum densities in Landau-Lifshitz's prescription associated with the monopole metric (26), we evaluate the required components of Υ^{ikjl} (14);

$$\begin{aligned} \Upsilon^{1002} &= \Upsilon^{2001} = -\Upsilon^{0102} = \frac{8\pi G\eta^2 xy(8\pi G\eta^2 - 1)}{r^2} \\ \Upsilon^{1003} &= \Upsilon^{2003} = -\Upsilon^{3001} = \frac{8\pi G\eta^2 xz(8\pi G\eta^2 - 1)}{r^2} \\ \Upsilon^{3002} &= -\Upsilon^{0203} = \frac{8\pi G\eta^2 yz(8\pi G\eta^2 - 1)}{r^2} \\ \Upsilon^{1003} &= \Upsilon^{3001} = \frac{mxz(m+1)}{r^2} \\ \Upsilon^{0202} &= \frac{(r^2 - 8\pi G\eta^2 y^2)(8\pi G\eta^2 - 1)}{r^2} \end{aligned} \tag{31}$$

Using these components in (13), we get the LL energy and momentum densities for monopole metric as follows,

$$\mathcal{L}_0^0 = \frac{G\eta^2(8\pi G\eta^2 - 1)}{r^2}$$
$$\mathcal{L}_1^0 = \mathcal{L}_2^0 = \mathcal{L}_3^0 = 0 \tag{32}$$

(iv) *Møller Energy Momentum Distribution for Monopole Metric*:
Using the metric in (26), we have found the required components of $\lambda_\mu^{\nu\alpha}$ are zero and we get the Møller energy and momentum densities for monopole as follows,

$$M_0^0 = 0$$
$$M_1^0 = M_2^0 = M_3^0 = 0 \tag{33}$$

(v) *Papapetrou Energy Momentum Distribution of Monopole Metric*:
Using the line element (26), we have found the required components of $N^{\mu\nu\alpha\beta}$ (22);

$$N^{1010} = \frac{8\pi G\eta^2 xy}{r^2} = -N^{0012}$$
$$N^{2030} = N^{3020} = \frac{8\pi G\eta^2 yz}{r^2} = -N^{0012}$$
$$N^{0022} = \frac{2r^2 - 8\pi G\eta^2(r^2 + y^2)}{r^2} \tag{34}$$
$$N^{0023} = -\frac{8\pi G\eta^2 yz}{r^2}$$

Using these components in (21) we obtain Papapetrou energy-momentum distribution for monopole metric,

$$\Sigma_0^0 = \frac{G\eta^2}{r^2}$$
$$\Sigma_1^0 = \Sigma_2^0 = \Sigma_3^0 = 0 \tag{35}$$

Also all energy-momentum densities for monopole metric can be seen in Table 1.

4 Discussion

In this study we have used the some of the topological defects metrics namely texture and monopole and calculated the energy-momentum densities of these metrics by using the some well-known definitions of the Einstein, Bergmann-Thomson, Landau-Lifshitz, Møller and Papapetrou definitions in GR theory. For this purpose, firstly we have transformed the texture and monopole metrics to Cartesian from spherical coordinates. Because the energy-momentum complexes (Einstein, Bergmann-Thomson, Tolman, Papapetrou, LL, Weinberg, Goldberg, Quadir-Sharif) give meaningful results when we transform the line element in quasi-Cartesian coordinates [10]. In this paper we have found that Einstein, Bergmann-Thomson, LL, Papapetrou energy-momentum distributions and the Møller momentum densities are well defined and different from zero except the Møller energy density ($M_0^0 = 0$)

Table 1 The energy and momentum densities for monopole metric

Prescription	Energy density	Momentum density
Berg.-Thoms.	$\Xi_0^0 = \frac{G\eta^2}{r^2}$	$\Xi_1^0 = \Xi_2^0 = \Xi_3^0 = 0$
Einstein	$\S_0^0 = \frac{G\eta^2}{r^2}$	$\S_1^0 = \S_2^0 = \S_3^0 = 0$
Papapetrou	$\Sigma_0^0 = \frac{G\eta^2}{r^2}$	$\Sigma_1^0 = \Sigma_2^0 = \Sigma_3^0 = 0$
Landau-Lifs.	$\mathcal{L}_0^0 = \frac{G\eta^2(\Lambda-1)}{r^2}$	$\mathcal{L}_1^0 = \mathcal{L}_2^0 = \mathcal{L}_3^0 = 0$
Møller	$M_0^0 = 0$	$M_1^0 = M_2^0 = M_3^0 = 0$

for texture metric. Also Einstein and Bergmann-Thomson energy definitions are the same. These results are agree with the results of the Aygün and Yilmaz [29]. They have investigated the Møller energy complexes of monopoles and textures in general relativity and teleparallel (the tetrad theory of gravity) gravity and found that although Møller energy definitions in GR and teleparallel gravity are different, energy distribution is the same and zero in both of these different gravitation theories in spherical coordinates. From these results, we can see that again the energy and momentum complex of Møller gives the possibility to perform the any coordinate system.

Although the energy momentum definitions for texture metric give different results, we found that same and acceptable results using the Einstein, Bergmann-Thomson and Papapetrou energy momentum definitions for monopole metric. However Landau-Lifshitz definition do not give the same result with these definitions and we found the Møller energy and momentum densities are zero like the previous work [29]. Also all momentum densities of the definitions are same and zero for monopole metric. We see that the η constant has a important role on the energy distributions for monopole metric in GR. If we take $\eta^2 = 0$ in (26), we obtain Minkowski metric from monopole metric and using this condition and from (28), (30), (32), (33) and (35) we evaluate Einstein, Bergmann-Thomson, LL, Møller and Papapetrou energy-momentum distributions are zero for Minkowski metric as folllows;

$$\S^0_{\alpha_M} = \Xi^0_{\alpha_M} = \Sigma^0_{\alpha_M} = \mathcal{L}^0_{\alpha_M} = M^0_{\alpha_M} = 0, \quad \alpha = 0, 1, 2, 3$$

Here lower indice M represents the Minkowski space-time. These results are agree with the previous results.

References

1. Einstein, A.: Preuss. Akad. Wiss. Berlin **47**, 778 (1915)
2. Tolman, R.C.: Relativity, Thermodynamics and Cosmology. Oxford University Press, London (1934)
3. Papapetrou, A.: Proc. R. Ir. Acad. A **52**, 11 (1948)
4. Landau, L.D., Lifshitz, L.D.: The Classical Theory of Fields. Addison-Wesley, Reading (1951)
5. Bergmann, P.G., Thomson, R.: Phys. Rev. **89**, 400 (1953)
6. Møller, C.: Ann. Phys. (NY) **12**, 118 (1961)
7. Weinberg, S.: Gravitation and Cosmology: Principles and Applications of General Theory of Relativity. Wiley, New York (1972)
8. Mikhail, F.I., Wanas, M., Hindawi, A., Lashin, E.I.: Int. J. Theor. Phys. **32**, 1627 (1993)
9. Vargas, T.: Gen. Relativ. Gravity **36**, 1255 (2004)
10. Virbhadra, K.S.: Phys. Rev. D **42**, 2919 (1990)
11. Radinschi, I.: Mod. Phys. Lett. A **15**, 2171 (2000)
12. Aygün, S., Baysal, H., Tarhan, İ.: Int. J. Theor. Phys. **46**, 2607 (2007)
13. Aygün, S., Aygün, M., Tarhan, İ.: Acta Phys. Pol. B **37**, 2781 (2006)
14. Aygün, M., Yilmaz, İ., Aygün, S.: Acta Phys. Pol. B **37**, 2795 (2006)

15. Vagenas, E.C.: Int. J. Mod. Phys. A **18**, 5949 (2003)
16. Vagenas, E.C.: Int. J. Mod. Phys. D **14**, 573 (2005)
17. Vagenas, E.C.: Mod. Phys. Lett. A **21**, 2006 (1947)
18. Radinschi, I.: Acta Phys. Slov. **49**, 789 (1999)
19. Yang, C.-I., Radinschi, I.: Chin. J. Phys. **41**, 326 (2003)
20. Salti, M., Havare, A.: Int. J. Mod. Phys. A **20**, 2165 (2005)
21. Xulu, S.S.: Int. J. Theor. Phys. **39**, 1153 (2000)
22. Gad, R.M.: Mod. Phys. Lett. **19**, 2004 (1847)
23. Xulu, S.S.: Int. J. Mod. Phys. A **15**, 2979 (2000)
24. Virbhadra, K.S.: Phys. Rev. D **60**, 104041 (1999)
25. Rosen, N., Virbhadra, K.S.: Gen. Relativ. Gravity **25**, 429 (1993)
26. Virbhadra, K.S., Parikh, J.C.: Phys. Lett. B **317**, 312 (1993)
27. Radinschi, I.: Chin. J. Phys. **39**, 231 (2001)
28. Xulu, S.S.: Astrophys. Space Sci. **283**, 23 (2003)
29. Aygün, M., Yilmaz, İ.: Chin. Phys. Lett. **24**, 874 (2007)
30. Vilenkin, A., Shellard, E.P.S.: Cosmic Strings and Other Topological Defects. Cambridge University Press, Cambridge (1994)
31. Xulu, S.S.: Int. J. Theor. Phys. **37**, 1773 (1998)